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RESEARCH REPORT
N° 8631
November 2014
Project-Team FUN

ISSN 0249-6399 ISRN INRIA/RR--8631--FR+ENG

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Research Report n° 8631 — November 2014 — 10 pages

Abstract: In this report, we study the ambiguity of source localization using signal processing of large aperture antenna arrays under spherical wave propagation. This novel localization approach has been recently proposed, providing an estimate of the source position by means of two methods: geometrical and analytical. The former finds the source position as the estimate of circular loci, the latter as a solution of a linear system of equations. Although this method is proved to work for a general array geometry, we show that it suffers from ambiguities for a particular class of array geometries. Namely, in 2D, we prove that when the array geometry is linear or circular, there exist two possible solutions where only one corresponds to the actual position of the source. We also prove a relation of symmetry between the solutions with respect to the array geometry. This relation is very useful to assist the disambiguation process for discounting one of the estimates. By extension to 3D, planar (resp. spherical) arrays exhibit the same behavior i.e they provide two symmetrical estimates of the source position when the latter is not on the array plane (resp. sphere).

Key-words: Location ambiguities, array processing, localization, spherical wave propagation

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Sur l’ambiguité de la localisation par réseaux d’antennes à large ouverture

Résumé : Dans ce rapport, nous étudions l’ambiguïté d’une approche de localisation à base de traitement du signal de réseaux d’antennes à large ouverture sous propagation ondulatoire sphérique. Cette approche de localisation a été récemment proposée. Elle fournit une estimation de la position de la source au moyen de deux méthodes: géométrique et analytique. La première méthode localise la source à l’intersection de cercles. La seconde, permet de calculer la position de la source en résolvant un système d’équations linéaires. Bien que cette méthode ait été prouvée dans le cas général, nous démontrons qu’elle souffre d’ambiguïtés pour une classe particulière de géométrie de réseaux d’antennes. À savoir, on montre qu’en 2D, lorsque la géométrie du réseau est linéaire ou circulaire, il existe deux solutions possibles où seulement une correspond à la position réelle de la source. Nous prouvons aussi une relation de symétrie entre les solutions par rapport à la géométrie du réseau. Cette relation est très utile pour aider le processus de localisation à éliminer l’une des estimations. Par extension au 3D, les réseaux planaires (resp. sphériques) présentent le même comportement c.a.d ils fournissent deux estimations symétriques de la position de la source lorsque celle-ci n’est pas sur le plan (resp. la sphère) du réseau .

Mots-clés : Ambiguïtés, réseaux d’antennes, traitement du signal, localisation, propagation sphérique

1 Introduction

Large aperture array (LAA) localization is a novel range-based localization technique that has been recently introduced in [1, 2]. The LAA approach is split in two consecutive phases: *Association* phase and then *Metric-Fusion* phase. In the former, ratio of ranges with respect to the antenna array elements are estimated from the eigenvalues of the signal covariance matrices. In the latter, those ratios (called metrics) are then fused to infer the position of the source as the intersection of circular loci built from the antenna array geometry and the estimated metrics. The LAA localization procedure was proved to overcome traditional range-based techniques using RSS, ToA/TDoA or AoA metrics [1, 2]. In this work, we rather focus on the *Metric-Fusion*, and we prove that it suffers from location ambiguities that are due to some singular array geometries. Namely, we prove that when the array geometry is linear (planar in 3D) or circular (spherical in 3D), location ambiguities appear. We also prove that there exist two possible solutions that are inverse with respect to the array geometry. Numerical simulation results are also provided to illustrate the problem in 2D.

The rest of this paper is organized as follows. The LAA localization procedure is summarized in Section 2. Location ambiguities are then proven in Section 3. Then, simulation results are shown in Section 4. Finally, conclusions are drawn in Section 5

2 LAA Localization Procedure

This section provides a summary of the theoretical framework that is required to achieve localization using antenna arrays. This framework was developed by Manikas *et al.* in [1, 2]. Consider a fully calibrated large aperture sparse array of N omnidirectional antennas (sensors, transceivers...etc), with a common reference point (zero-phase reference point taken to be the origin of the coordinate system). The array antenna locations are known and defined by the matrix $\mathbf{r} \in \mathcal{R}^{3\times N}$ with respect to the system origin, that is

$$\mathbf{r} = \left[\underline{r_1}, \underline{r_2}, \ldots, \underline{r_N}\right] = \left[\underline{r}_x, \underline{r}_y, \underline{r}_z\right]^T \tag{1}$$

where $\underline{r_i} \in \mathcal{R}^{3\times 1}$ for $i = 1 \ldots N$ is the location of i^{th} antenna in the array and $\underline{r}_x, \underline{r}_y, \underline{r}_z \in \mathcal{R}^{N\times 1}$ denote the x, y and z coordinates of the N antennas. The array aperture is therefore given by

$$D = \max_{\forall i,j} \left\|\underline{r}_i - \underline{r}_j\right\| \tag{2}$$

Assume the array operates in the presence of a single transmitting source, with a carrier frequency F_c, and located at an unknown position[1] with respect to the array reference point

$$\underline{r}_m = [x, y, z]^T = \rho \cdot \underline{u}(\theta, \phi). \tag{3}$$

The vector $\underline{u}(\theta, \phi) = [\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi]^T$ denotes the unity norm vector pointing in the direction of the source. The LAA localization procedure is carried out in two consecutive phases: *Association* phase and *Metric-Fusion* phase. In *Association* phase, the signals received by the array are used to estimate spatial relationships between the array elements and the source. Such *metrics* are then used in the *Metric-Fusion* phase by a dedicated algorithm, to compute the position of the source.

[1]The azimuth angle θ is measured anticlockwise with respect to the positive x-axis, and the elevation angle ϕ is measured anticlockwise from the x-y plane.

2.1 Association Phase

The LAA localization estimates the following *metrics*

$$\mathcal{K}_i = \frac{\rho_i}{\rho_1} = \frac{\|\underline{r}_m - \underline{r}_i\|}{\|\underline{r}_m - \underline{r}_1\|} \quad \forall i = 2 \ldots N, \tag{4}$$

where $\mathcal{K}_i$ is estimated from the signals collected from the array when the first element is at the *primary* reference point [1].

2.2 Metric-Fusion Phase

The ratio $\mathcal{K}_i$ in (4) is mathematically known as the Apollonius circle of points $\underline{r}_1$ and $\underline{r}_i$ [3, §18.3]. This leads to the following theorem proven in [1].

Theorem 1 (LAA localization -*geometrical*)**.** *Given a N-element array, one can estimate $N-1$ metrics $\mathcal{K}_i$ taken with respect to $N-1$ different* array reference points*, where the source location $\underline{r}_m$ is at the common intersection of the $N-1$ circular loci which centers $\underline{r}_{c_i}$ and radii R_{c_i} are defined for $i = 2 \ldots N$ by*

$$\underline{r}_{c_i} = \frac{1}{1-\mathcal{K}_i^2}\underline{r}_i - \frac{\mathcal{K}_i^2}{1-\mathcal{K}_i^2}\underline{r}_1 \tag{5}$$

$$R_{c_i} = \left|\frac{\mathcal{K}_i}{1-\mathcal{K}_i^2}\right| \cdot \|\underline{r}_1 - \underline{r}_i\| \tag{6}$$

Hence, in $\mathcal{R}^2$ space, a minimum of 4 sensors (3 loci) is required for an unambiguous position estimate of the source location; and in $\mathcal{R}^3$ space a minimum of 5 sensors (4 loci) will be required. In [1], (4) is derived to construct a set of linear equations as follows.

Theorem 2 (LAA localization -*analytical*)**.** *In the general $\mathcal{R}^3$ case, the source location can be obtained by solving the following set of equations $\mathbb{H}\underline{r}'_m = \underline{b}$:*

$$\underbrace{\begin{bmatrix} 2(\underline{r}_1 - \underline{r}_2)^T, & (1-\mathcal{K}_2^2) \\ 2(\underline{r}_1 - \underline{r}_3)^T, & (1-\mathcal{K}_3^2) \\ \vdots & \vdots \\ 2(\underline{r}_1 - \underline{r}_N)^T, & (1-\mathcal{K}_N^2) \end{bmatrix}}_{\mathbb{H}} \underbrace{\begin{bmatrix} \underline{r}_m \\ \rho_1^2 \end{bmatrix}}_{\underline{r}'_m} = \underbrace{\begin{bmatrix} \|\underline{r}_1\|^2 - \|\underline{r}_2\|^2 \\ \|\underline{r}_1\|^2 - \|\underline{r}_3\|^2 \\ \vdots \\ \|\underline{r}_1\|^2 - \|\underline{r}_N\|^2 \end{bmatrix}}_{\underline{b}}. \tag{7}$$

Figure 1 shows an illustration of the *Metric-Fusion* phase with our implementation using the array geometry in [1, sec. IV.A].

3 LAA Localization Ambiguities

It is clear from (4) and Theorem 1 that the circular loci configuration depends on the array geometry. In this section, we will prove that location ambiguities exist for two types of array geometries (circular and linear) even when the number of antenna elements N is higher than the minimum required. First, let us recall circle inversion [3, §20.1] under the following definitions for consistency with the addressed problem.

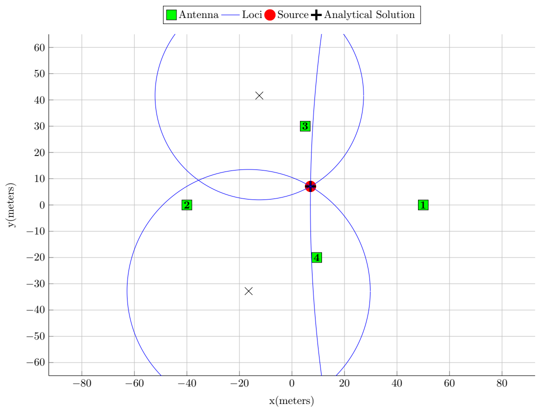


Figure 1: LAA Localization example (consistent with [1, sec. IV.A])

Definition 1 (Line-Circle)**.** *A circle is uniquely defined by two points* $(\underline{r}_1, \underline{r}_i)$ *and a constant ratio* $\mathcal{K}_i$*.*

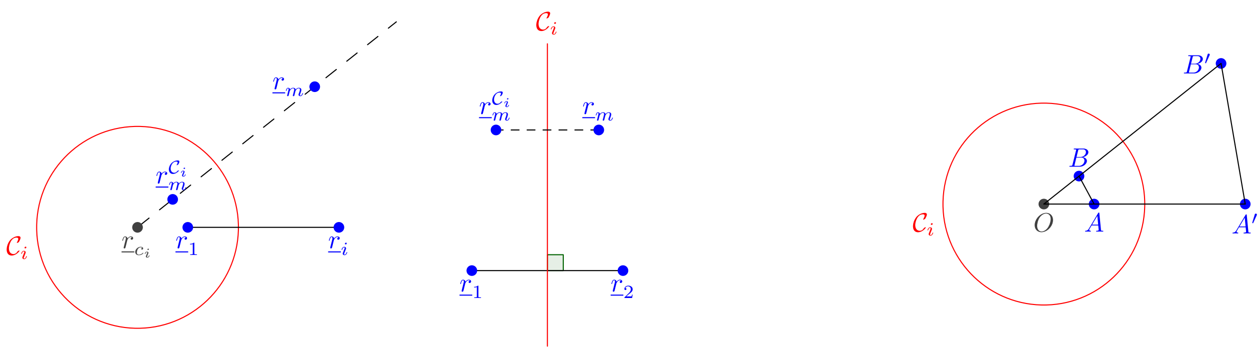
$$\mathcal{C}_i = \mathcal{C}(\underline{r}_1, \underline{r}_i, \mathcal{K}_i) \triangleq \mathcal{C}(\underline{r}_{c_i}, R_{c_i}).$$

When $\mathcal{K}_i = 1$*, the circle degenerates to a line that is the perpendicular bisector of the segment* $(\underline{r}_1 - \underline{r}_i)$*.*

Definition 2 (Inversion with respect to a Circle)**.** *Given a point* $\underline{r}_m$ *and a line-circle* $\mathcal{C}_i$*. The inverse of* $\underline{r}_m$ *with respect to* $\mathcal{C}_i$ *is given by the* $(\cdot)^{\mathcal{C}_i}$ *operator*

$$\underline{r}_m^{\mathcal{C}_i} = \underline{r}_{c_i} + \frac{R_{c_i}^2}{\|\underline{r}_m - \underline{r}_{c_i}\|^2} \cdot \left(\underline{r}_m - \underline{r}_{c_i}\right).$$

The center of $\mathcal{C}_i$ *is called the center of inversion. It appears that when* $\mathcal{K}_i = 1$*,* $\underline{r}_m^{\mathcal{C}_i}$ *becomes the reflection of* $\underline{r}_m$ *across the perpendicular bisector of the segment* $\|\underline{r}_1 - \underline{r}_i\|$ *(cf. [3, §20.3]).*



(a) With respect to a real circle (b) With respect to a line (c) Property *P.5*

Figure 2: Circle Inversion

The circle inversion has many interesting mathematical properties, but we will recall the following ones.

P.1: The circle inversion is an involution i.e $(\underline{r}_m^{\mathcal{C}_i})^{\mathcal{C}_i} = \underline{r}_m$.

P.2: Inverting the center of the inversion sends it on ∞.

P.3: All points on the inversion circle are invariant under inversion.

P.4: The inverse of any point inside the inversion circle lies outside it and vice-versa.

P.5: Given a triangle OAB, a real circle $\mathcal{C}_i$ centered in O, and $A' = A^{\mathcal{C}_i}, B' = B^{\mathcal{C}_i}$, then the triangles OAB and $OA'B'$ are similar [3, §23.1].

Circle inversion is illustrated in Figure 2 for real and degenerate circles. The following proposition is fundamental for proving the existence of ambiguities in the localization process.

Proposition 1. *Given a line-circle $\mathcal{C}_i$, and two distinct points A and B on $\mathcal{C}_i$. Let $\mathcal{C}_j = \mathcal{C}(A, B, \mathcal{K}_j)$, then the following statement holds for every point M in the plane:*

$$M \in \mathcal{C}_j \Longleftrightarrow M' = M^{\mathcal{C}_i} \in \mathcal{C}_j.$$

Proof. See Annex A. □

3.1 Ambiguities in 2D

An antenna array geometry $\mathbf{r}$ is said to be *linear* when all its elements $\underline{r}_i$ are *collinear* i.e they belong to the same line. The array geometry is said to be *circular* when all its elements $\underline{r}_i$ are *concyclic* i.e they belong to the same circle. Since a line is a degenerate circle ($\mathcal{K}_i = 1$), the following result is valid for both linear and circular array geometries.

Theorem 3. *When performing LAA localization with a linear (or circular) array $\mathbf{r}$ under the presence of a source $\underline{r}_m$, the resulting circular loci $\mathcal{C}_i = \mathcal{C}(\underline{r}_1, \underline{r}_i, \mathcal{K}_i)$ have two intersecting points $\underline{r}_m$ and $\underline{r}_m^{\mathcal{C}_r}$ where $\mathcal{C}_r$ is the line-circle passing through the array elements.*

Proof. Theorem 1 states that $\underline{r}_m \in \mathcal{C}_i \forall i = 2 \dots N$. According to Proposition 1, we have $\underline{r}_m^{\mathcal{C}_r} \in \mathcal{C}_i \forall i = 2 \dots N$. Therefore, the loci will be intersecting in two points that are inverse with respect to the array $\mathcal{C}_r$:

$$\bigcap_{i=2\dots N} \mathcal{C}_i = \{\underline{r}_m, \underline{r}_m^{\mathcal{C}_r}\}$$

□

Corollary 1. *When the source is on the array line-circle $\mathcal{C}_r$ there is exactly one solution (property P.3 of the circle inversion). In the case of a circular array, when the source in on the array center, its inverse is sent to ∞, we therefore have one and only one finite solution.*

3.2 Ambiguities in 3D

Since circle inversion can be generalized to sphere in inversion in three dimensions [4, §5.1], all above results remain valid in 3D. Circle inversion becomes sphere inversion, whereas reflection across a line becomes reflection across a plane. Also, the ambiguities shall concern planar and spherical array geometries. That is, ambiguities appear when the source is not on the array plane for the planar case, and not on the array sphere for the spherical case.

3.3 Ambiguities in the Analytical solution

In Theorem 2, a system of equations is built to compute the estimate of the position using the pseudo-inverse matrix of $\mathbb{H}$:

$$\underline{r}'_m = \mathbb{H}^{\#}\underline{b}. \tag{8}$$

For this to work, the matrix $\mathbb{H}$ shall have a rank of 3 in 2D and a rank of 4 in 3D. Simulations reveal that under ambiguous array configuration, the matrix $\mathbb{H}$ has a rank of 2 in 2D and 3 in 3D, thus leading to an erroneous estimation. We leave the mathematical proof of this statement to a future work.

4 Numerical results

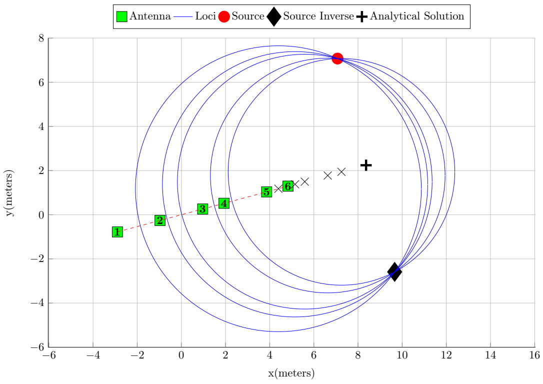
In order to verify the ambiguities and the symmetry of the solutions, we have selected two arrays that are respectively linear (passing through the origin), and circular (centered in the origin).

$$\begin{aligned}
\mathbf{r}_1 &= \begin{bmatrix} \cos(15°) & -\sin(15°) & 0 \\ \sin(15°) & \cos(15°) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & -1 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\mathbf{r}_2 &= \begin{bmatrix} 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & 2 & -2 & -2 & 2\sqrt{2} & -2\sqrt{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{9}$$

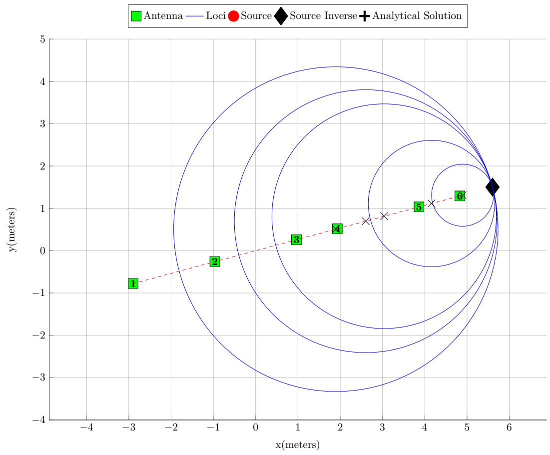
The results are shown in Figure 3. The existence of two solution that are inverse with respect to the array circle (or symmetrical with respect to the array line) is clearly verified. It is interesting to observe, in Figure 3e how the circular loci degenerate to lines when the source is on the array center, sending the second solution to ∞.

5 Conclusion

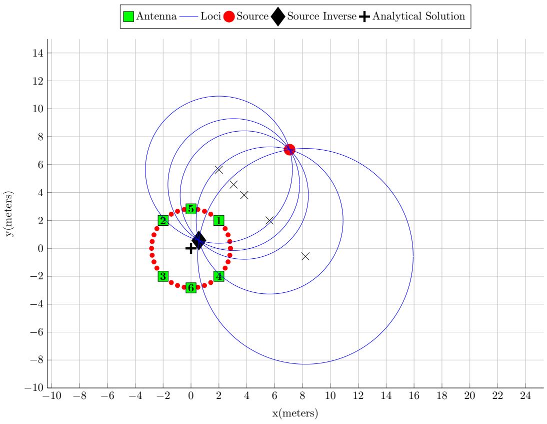
In this report, we have shown the existence of location ambiguities when using LAA Localization for linear (planar in 3D) and circular (spherical in 3D) array geometries. We have proved that, in general, there are two solutions to the location problem that are inverse with respect to the array line (plane) or circle (sphere). This relationship is essential to assist the disambiguation of the solution. With noisy measurements, the circular loci do not have exact intersections, but rather clusters of intersecting points. By using such geometries, the inversion relationship between the cluster points can be used to efficiently discount one of the solutions.



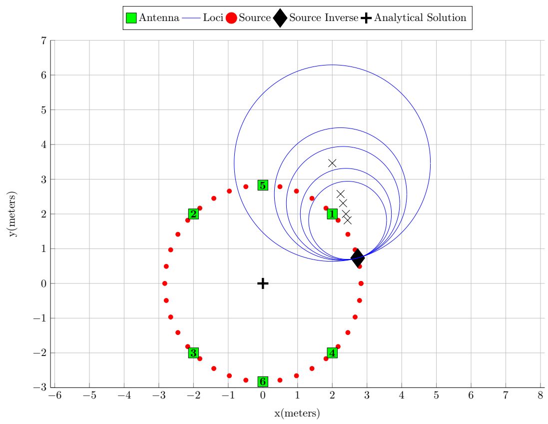
(a) Linear array with two inverse solutions



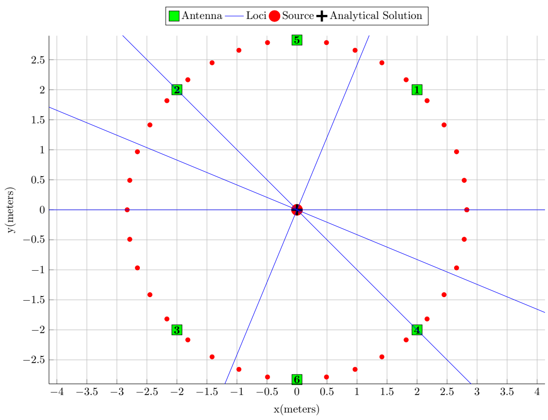
(b) Linear array with two coinciding inverse solutions



(c) Circular array with two inverse solutions



(d) Circular array with two coinciding inverse solutions



(e) Circular array with one solution at ∞

Figure 3: LAA Location ambiguities in 2D

Appendices

A Proof of Proposition 1

Let us first prove the following Lemma.

Lemma 1. *Given a line-circle $\mathcal{C}_i$, and two distinct points A and B on $\mathcal{C}_i$. Let M a point of the plane, distinct from O, with its inverse $M' = M^{\mathcal{C}_i}$, then following ratio is always verified:*

$$\frac{MA}{MB} = \frac{M'A}{M'B}.$$

Proof. According to the properties $P.3$ and $P.5$ of the circle inversion, the triangle OAM (resp.OBM) is similar to OAM' (resp. OBM'). Therefore

$$\frac{MA}{M'A} = \frac{OM}{OM'} \text{ and } \frac{MB}{M'B} = \frac{OM}{OM'}.$$

By definition of the circle inversion we have $OM \cdot OM' = R^2$, leading to

$$MA = \left(\frac{OM}{R}\right)^2 \cdot M'A \text{ and } MB = \left(\frac{OM}{R}\right)^2 \cdot M'B.$$

Which proves the lemma. Note that this ratio, independent from the circle radius, is still valid when the circle degenerates to a line. □

According to Lemma1 we have $\frac{MA}{MB} = \frac{M'A}{M'B}$. Therefore M and M' belong to the same line-circle $\mathcal{C}(A, B, \mathcal{K}_j)$. Note that when $\mathcal{C}_i$ is a line, such property is trivial to prove by using elementary properties of line reflection.

Acknowledgment

This work is partially supported by a grant from CPER Nord-Pas-de-Calais / FEDER Campus Intelligence Ambiante, the Inria Project Lab CityLab@Inria and the FP7 SMART CITIES VITAL.

The authors thank Prof. A. Manikas and Dr. M. Willerton for corresponding on the ambiguity issue in this report.

References

[1] A. Manikas, Y. I. Kamil, and M. Willerton, "Source localization using sparse large aperture arrays," *Signal Processing, IEEE Transactions on*, vol. 60, no. 12, pp. 6617–6629, 2012.

[2] A. Manikas, Y. I. Kamil, and P. Karaminas, "Positioning in wireless sensor networks using array processing," in *Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE*, pp. 1–5, IEEE, 2008.

[3] D. Pedoe, *Geometry: A comprehensive course.* Courier Dover Publications, 2013.

[4] D. E. Blair, *Inversion theory and conformal mapping*, vol. 9. American Mathematical Soc., 2000.

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Publisher
Inria
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ISSN 0249-6399